User Selection in MIMO Interfering Broadcast Channels

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***Abstract*—This paper tries to address two issues one is selection of users(or user subset) in an interference system which maximizes the sum rate and the other is extending the former to interference broadcast channel[IFBC]. IFBC is different from interference channel on the part that interference channel[IFC] contain single user so suffer from inter channel interference(ICI) only whereas interference broadcast channel[IFBC] incorporates communication between multiple users(cellular network) resulting in interference among user called inter user interference(IUI) as well as among different channels causing inter channel interference(ICI). MMSE and other conventional approaches tries to mitigate the interference which reduces the diversity order and degree of freedom so we come up with the idea of interference alignment(IA). IA utilizes the mutual information available among users and help in combating ICI and IUI by designing the appropriate beamformer which can align the whole interference in on subspace. To implement this author has presented two algorithms which achieves a much lower computational complexity than brute force algorithm with a slight compromise on sum rate.**

I. Introduction

MIMO wireless systems have the capability to achieve very high sum rates. The presence of multiple transmit and receive antennas on multiple users system results in the interference. Numerous papers has been published in this domain recently to combat high interference and to achieve maximum degree of freedom(dof) in interference networks. One way which is exclusively opted for improving the dof in MIMO wireless system is **Interference Alignment(IA)**. IA is different from other conventional techniques(ZF or MMSE) in way that it align the interference rather than mitigating it. Conventional communication is based upon thinking that in interference channel one user don't have any information about other user. So it is optimum to be greedy and maximize its own rate. In such a system maximum sum rate for MIMO interference channel is equivalent to single user rate communication link. Some research groups has shown that the sum rate can be made to increase linearly with number of users by sharing some information among them. This is called as Interference Alignment. IA is a linear precoding technique which aims to align interfering signal in along (any dimension available time, frequency, space etc.) spatial dimension facilitated by numbers of transmit or receiver antennas.The heart of IA lie in the fact that after precoding the interference signal must lie in the minimal dimension. IA results in an information theoretic output of K/2 (K is the number of transmit antennas)i.e rate upto atleast have of channel capacity is achievable or we can say that half of the users will be able to achieve channel capacity. In this paper we are addressing majorly the problem of sum rate maximization using the proper user subset selection through two sub-optimal algorithms o-algorithm(orthogonalization based algorithm) and s-algorithm(sum rate based algorithm).These algorithms will be extended to Interference Broadcast channel[IFBC]. IFBC is interference cellular channel with a large number of users specially in downlink causing interference among users [IUI] and interference amongst channel[ICI]. The best way to choose a user subset with maximum rate is brute force algorithm(try all possibles subsets) which is exponentially complex so we proposed two sub-optimal algorithm of linear complexity with a small compromise on sum rate. These algorithms exploits the two properties of co-ordinate ascent approach and orthogonality between the space spanned by users' signal and interference channel space. IA is difficult in implementation as the effective channel of one user is sensitive to variation in the effective channel of any other user. This sensitivity results in extended grouping done for achieving IA results in effective channel taking a special structure due to which the user's effective channel matrix relates itself very closely with the adjacent BS channel.

The IA was implemented by [3] for K-transmitter and K-receiver time varying interference channel. A number of papers have shown that IA is implementable for Equal transmitter and receiver configuration for constant channel and a close form solution is available for precoder to achieve IA with global channel knowledge at each node. In above paper while implementing for IA they didn't take sum rate into consideration. So [5] gave an iterative expression for K=3 in order to implement IA and achieving maximum sum-rate for a MIMO system with local channel information. A number of papers have been in literature for IFBC and specially for IFC to combat interference using conventional approaches like MMSE and ZF. These approaches didn't show much improvement in sum rate specially for overloaded systems. IA to some level proved efficient but it requires the sharing of information which can become an excessive load over the network.

II. System Model and Background

In the paper we will considered following nomenclature

$y_k^{[l]}$: signal received by k^{th} user in l^{th} cell $\in \mathbb{C}^{N\times d_s}$

$x_k^{[l]}$: signal intended for k^{th} user in l^{th} cell by BS $\in \mathbb{C}^{M\times d_s}$

$H_k^{[l,j]}$: channel matrix from j^{th} BS to k^{th} user in l^{th} cell

$U_k^{[l]}$: receive beam-forming matrix for k^{th} user in l^{th} cell

$V_k^{[l]}$: Pre-coding matrix for k^{th} user in l^{th} cell

$s_k^{[l]}$: $d_s \times 1$ symbol vector with i^{th} symbol as $s_{k,i}^{[l]}$.

The maximum number of stream can be transmitted in parallel is given d_s = max(M,N).The transmitter power for l_{th} BS is limited by $\mathbb{E}\{\sum_{k=1}^{K}||x_k^{[l]}||\} \leq P_l$ The signal to be transmitted is precoded using $V_k^{[l]} \in \mathbb{C}^{M\times d_s}$ as

$$x_k^{[l]} = \sum_{i=1}^{d_s} v_{l,i}^{[k]} s_{k,i}^{[l]} = V_k^{[l]} s_k^{[l]} \tag{1}$$

$$y_k^{[l]} = \sum_{j=1}^{L} H_k^{[l,j]} \sum_{i=1}^{L} x_k^{[j]} \tag{2}$$

$$= \underbrace{H_k^{[l,l]} V_k^{[l]} s_k^{[l]}}_{Desiredsignal} + \underbrace{\sum_{j=1,i\neq k}^{L} H_k^{[l,l]} V_i^{[l]} s_i^{[l]}}_{Inter-userInterference}$$

$$+ \underbrace{\sum_{j=1,j\neq l}^{L} \sum_{i=1}^{K} H_k^{[l,j]} V_i^{[j]} s_i^{[j]} + n_k^{[l]}}_{Inter-cellInterference} \tag{3}$$

where the receive beamformer is $U_k^{[l]} \in \mathbb{C}^{N\times d_s}$ is used on receive side after signal passes through a Rayleigh flat fading slow varying channel $H_k^{[l,j]} \in \mathbb{C}^{N\times M}$ with each component independent and identically distributed and having unit variance.This paper will be implemented under some reasonable **assumptions** of considering the channel to be affected by IID zero mean and σ^2 variance gaussian noise, reciprocal in nature and with number of receive antennas to be more than the number of transmit antennas(not overloaded).The receiver equation after beamforming can be written as:

$$\tilde{y}_k^{[l]} = U_k^{[l]H} H_k^{[l,l]} V_k^{[l]} s_k^{[l]} + U_k^{[l]H} (\sum_{i=1,i\neq k}^{K} H_k^{[l,l]} V_i^{[l]} s_i^{[l]})$$

$$+ U_k^{[l]H} (\sum_{j=1,j\neq l}^{L} \sum_{i=1}^{K} H_k^{[l,j]} V_i^{[j]} s_i^{[j]}) + \tilde{n}_k^{[l]} \tag{4}$$

in this equation first term represents the desired signal, second term represents the inter-user interference,third term represents the inter-channel interference and the last term denotes the noise after receiver decoding using receive decoding matrix U.

A. Conditions for Interference cancellation

In simple term the problem can be stated as:

$$\text{Avoiding IUI} \; : U_k^{[l]H} H_k^{[l,l]} V_k^{[l]} = 0; \text{ for all } i \neq j \tag{5}$$

$$\text{Avoiding ICI} \; : U_k^{[l]H} H_k^{[l,l]} V_k^{[l]} = 0; \text{ m=1,2....,K, for all } j \neq l \tag{6}$$

$$: \text{Rank}(U_k^{[l]H} H_k^{[l,l]} V_k^{[l]}) = d_s \tag{7}$$

$$: \mathbb{E}(\sum_{k=1}^{K} ||x_k||^2) < P_L \tag{8}$$

The first equation and second signal means that we need to

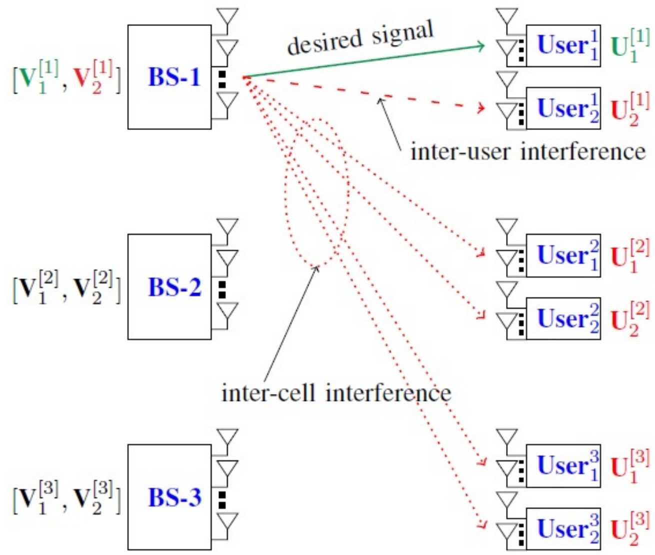

Fig. 1. MIMO-IFBC with L = 3 and K = 2 in each cell where the BS-1 is shown to be generating IUI and ICI for the users in its own cell and neighboring cells respectively.

design the beamformer which can project the signal in space which is orthogonal to the space spanned by the interference. Beamformer helps the channel to tale a new representation as stated below and this process result into noise becoming correlated which can be whitened using whitening filter $W_k^{[l]}$. This filter chosen is in such a manner that the resulting noise signal have zero mean and σ^2 variance

$$\begin{aligned} W_k^{[l]} =& (U_k^{[l]H} U_k^{[l]})^(1/2) \\ \bar{y}_k^{[l]} =& W_K^{[l]} (\bar{H}_K^{[l,l]} s_k^{[l]} + \tilde{n}_k^{[l]}) \\ \bar{y}_k^{[l]} =& W_K^{[l]} \bar{H}_K^{[l,l]} s_k^{[l]} + \bar{n}_k^{[l]} \end{aligned} \tag{9}$$

$$\begin{aligned} \mathbb{E}(\bar{n}_k^{[l]}) =& \mathbb{E}(W_K^{[l]} \tilde{n}_k^{[l]}) \\ =& W_K^{[l]} \mathbb{E}(\bar{n}_k^{[l]}) = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} \mathbb{E}(\bar{n}_k^{[l]} . \bar{n}^{[l]}) =& \mathbb{E}(W_K^{[l]} \tilde{n}_k^{[l]} . \tilde{n}_k^{[l]H} W_K^{[l]H}) \\ =& W_K^{[l]} \mathbb{E}(\tilde{n}_k^{[l]} . \tilde{n}_k^{[l]H}) W_K^{[l]H} \\ =& \sigma^2 W_K^{[l]} W_K^{[l]H} \\ =& \sigma^2 . I_{d_s} \end{aligned} \tag{11}$$

the above procedure results in the sum rate expression as

$$R = \sum_{l=1}^{L} \sum_{k=1}^{K} R_k^{[l]} \text{ using} \tag{12}$$

$$\begin{aligned} R_k^{[l]} =& log_2 |I_{ds} + W_k^{[l]} \bar{H}_k^{[l,l]} K_{xx} \bar{H}_k^{[l,l]H} W_k^{[l]H}| \text{ we get} \\ =& \sum_{l=1}^{L} max \sum_{k=1}^{K} log_2 |I_{ds} + W_k^{[l]} \bar{H}_k^{[l,l]} Q_k^{[l]} \bar{H}_k^{[l,l]H} W_k^{[l]H}| \end{aligned} \tag{13}$$

where $Q_k^{[l]} = \mathbb{E}(x_k^{[l]}.x_k^{[l]H})$ represents the input co-variance matrix for the k^{th} user in l^{th} cell the solution to this problem is waterfilling algorithm.

B. Extended Grouping Scheme

Grouping helps us in considering the whole group as a single entity which can be useful in aligning the interference space. Grouping can be achieved with a proper choice of receive beamformer $U_k^{[l]}$ for all the cell in such a way that the users in cell next to it can be clustered to align the interference from it in the same space as the previous cell.When this process is repeated over all the cells the ICI from all the BS spans the same subspace.

$$span[H_i^{l,l+1]H} U_i^{[l+1]} = span[H_{i+1}^{l,l+1]H} U_{i+1}^{[l+1]} \text{ for all i} \quad (14)$$

which results in ICI from all the BS spanning the same space defined as :

$$G_l = span[H_1^{l,l+1]H} U_1^{[l+1]}] = span[H_2^{l,l+1]H} U_2^{[l+1]}] \\ = ...span[H_K^{l,l+1]H} U_K^{[l+1]}] \quad (15)$$

this equation can be expressed in matrix form as

$$\begin{bmatrix} I_M & -H_1^{[l+1,l]H} & 0 & ... & 0 \\ I_M & 0 & -H_2^{[l+1,l]H} & ... & 0 \\ . & . & . & ... & 0 \\ I_M & 0 & 0 & ... & -H_K^{[l+1,l]H} \end{bmatrix} \begin{bmatrix} G_l \\ U_1^{[l+1]} \\ . \\ . \\ U_K^{[l+1]} \end{bmatrix}$$

$= F_l X_l = 0$ ....(16A)

- this matrix has rows = K.M
- and number of columns= M+K.N

resulting in a F_l having KM×(M+K×N) dimension whose null space contains the matrix X_l. Due to large dimensions of F_l its not logical to find the null space by Gram Schmidt procedure but we can exploit the sparsity of this matrix to reduce the complexity.The solution of above equation gives us the receive beamformers for all the BS's and grouping gives us the freedom for l^{th} BS to consider the next cell as a single ICI channel. Using the conditions of interference cancellation precoding matrices for users corresponding to $l^t h$ BS can be designed.

$$U_k^{[l]H} H_k^{[l,l]} V_k^{[l]} = 0; \text{ m=1,2....,K, for all } j \neq l \quad (16)$$

$$V_k^{[l]} \subset \text{null}(\underbrace{(U_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]H} H_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]H})}_{\text{effective ICI channels}}$$

$$\underbrace{(U_{t(t=1...K,\neq k)}^{[l)]H} H_{t(t=1...K,\neq k)}^{[l,l]H})}_{\text{effective IUI channels}} \underbrace{G_l}_{\text{effective interference channels}}) \quad (17)$$

III. User Selection

Consider the system that supports K users out of K_l available users in the cell.The sum rate can be maximized if we choose optimum user subsets for all the cells from the possible available subsets. We cannot perform search over the whole search range available as it may cause a very high computation burden for example a 2 cell and 3 user system which can accommodate 50 users will take 2x10^9 computations.
Consider the set of user in l_{th} cell: $T^{[l]}$ = 1,2,...,K_l
User subset $S^{[l]}$ for l_{th} cell having $|S^{[l]}|$ = K
For representation ease the sum rate expressions can be reformulated as

$$\mathcal{R}(S^{[1]},...., S^{[L]}) = \sum_{l=1}^{L} \sum_{k \in S^{[l]}} \mathcal{R}_k^{[l]} \quad (18)$$

$$\mathcal{R}_{opt} = \max_{s^{[l]} \subset \mathcal{T}^{[l]}} \mathcal{R}(S^{[1]}, S^{[2]}............, S^{[L]}) \quad (19)$$

A. Orthogonality approach

Search space of an algorithm is an important parameter to govern its computation complexity. In this section we will present an algorithm whose search space have been reduced intelligently by exploiting the orthogonality between signal and interference space. The algorithm is suboptimal as we tried to achieve an optimum trade of between sum rate and complexity. The approaches and algorithm we have for simple interference networks cannot be extended to IFBC because its not logical to compute the precoding and beamforming matrices for given channel matrix because this may be a very expensive procedure if the network has a very large size. So what will we do is we try to find how $V_k^{[l]}$ is related to effective channel matrix $H_k^{[l,l]H} U_k^{[l]}$. This result may help in eliminate the computation of $U_k^{[l]}$ and $V_k^{[l]}$.As we know from the previous equation that if channel takes the form $H^H.U$ then interference can be aligned. In downlink scenario the interference is defined from user viewpoint hence the channel can be defined as HV. This channel can be made interference free if we can make the channel of the form H^H and precoding matrix $U_k^{[l]}$. This problem can be addressed by the concept of network reciprocity.

1) Concept of Network reciprocity: It is equivalent to concept in network theory which if network holds reciprocity property then the transmitter can be replaced with receiver and vice-versa.
Condition for reciprocity : $\vec{H}_k^{[l,j]} = H_k^{[j,l]H}$ Reciprocity don't affect total power and interference from BS(because of grouping). If this condition is satisfied then N×1 transmit precoding matrix $\vec{V}_k^{[l]}$ and M×1 receive beamforming matrix $\vec{U}_k^{[l]}$ satisfies these equations :

$$\text{Avoiding IUI} \;: \vec{U}_k^{[l]H}\vec{H}_k^{[l,l]}\vec{V}_k^{[l]} = 0;\ \text{for all } i \neq j \tag{20}$$

$$\text{Avoiding ICI} \;: \vec{U}_k^{[l]H}\vec{H}_k^{[l,l]}\vec{V}_k^{[l]} = 0;\ \text{m=1,2....,K, for all } j \neq l \tag{21}$$

$$: \text{Rank}(\vec{U}_k^{[l]H}\vec{H}_k^{[l,l]}\vec{V}_k^{[l]}) = d_s \tag{22}$$

$$: \mathbb{E}(\sum_{k=1}^{K}||x_k||^2) < P_L \tag{23}$$

These conditions matches with the initial system interference conditions if we replace $\vec{U}_k^{[l]}$ by $V_k^{[l]}$, $\vec{V}_k^{[l]}$ by $U_k^{[l]}$ and take transpose. This is called as **Reciprocity of Alignment** which implies that if transmitter and receiver are exchanged system will remain unchanged. Reciprocity of alignment helps us to establish relation ship amongst $\vec{U}_k^{[l]}$ and $\vec{V}_k^{[l]}$ which can be stated as follows:

$$\vec{U}_k^{[l]} \subset \text{null}((\underbrace{\vec{H}_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]}\vec{V}_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]}}_{\text{effective ICI channels}}) \underbrace{(\vec{H}_{t(t=1...K,\neq k)}^{[l,l]H}\vec{V}_{t(t=1...K)}^{[l]})}_{\text{effective IUI channels}})^{\perp} \tag{24}$$

$$V_k^{[l]} \subset \text{null}[(\underbrace{(H_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]H}U_{t(t=1...K)}^{[s(s\neq l,\neq l+1)]})}_{\text{effective ICI channels}} \underbrace{(H_{t(t=1...K,\neq k)}^{[l,l]H}U_{t(t=1...K,\neq k)}^{[l)]})}_{\text{effective IUI channels}} \underbrace{G_l}_{\text{effective interference channels}})^{\perp} \tag{25}$$

These equations gives a good idea that the signal space which is closer to the the orthogonal space of interference will have a better projection over the space spanned by effective interference channel. This is good measure of finding a strong channel $\vec{U}_k^{[l]H}\vec{H}_k^{[l,l]}\vec{V}_k^{[l]}$ without calculating the $V_k^{[l]}$ and $U_k^{[l]}$. This closeness among the subspaces can be measured using Chordal distance.

2) Distance among subspaces: Chordal distance is basically defined in the **Grassmannian spaces**. Grassmannian space is set of all possible n-dimensional spaces which lies in m-dimensional euclidean space. Any m×n matrix is called a generator matrix for a n-dimensional Euclidean plain P∈G if all its columns span P. Say A_G and B_G are generator matrices with orthogonal columns then distance between P and Q is given by **Chordal Distance** between A_G and B_G defined as :

$$d_c(P,Q) = \frac{1}{\sqrt{2}}||A_G.A_G^H - B_G.B_G^H||_F \tag{26}$$

In Grassmannian space the degree of orthogonality between two subspaces in defined by their chordal distance.

3) Selection of Users: Concept of reciprocity and chordal distance form the basis of our paper. One more very beautiful equivalence which can be drawn from the above discussion. In this algorithm we are looking for a very strong channel for which we are trying to find high chordal distance user using Forbenius norm. Forbenius norm measures of energy so higher the distance stronger the channel. Hence K users will be selected on the basis of the Forbenius norn. If K is less than 3 we can use previous formula in equation (11) otherwise we have to go for some better approach used in reference [1]. We will define a matrix $S_k^{[l]temp}$ whose elemnts initially will be same as $S_k^{[l]}$. Then iteratively we will keep replacing k_{th} element $s_k^{[l]}$ with the j_{th} element $s_k^{[l]}$. Algorithm will approach users in co-ordinate ascent manner. A user will be selected if it has a high chordal distance and maximizes the sum rate. Any selected user will have to go through the process of Generator matrix computation which is nothing but a matrix containing column space as orthonormal basis function of the user matrix. The orthonormal basis function can be computed by using standard procedure of **Gram schmidt procedure**. This algorithm has been put in appendix-I for access. Interference alignment has been implemented in this on the part that all the Base stations has been sharing information amongst themselves for choosing the optimum user. This sharing of iformation helps in the rate to reach atleast half the channel capacity.

B. Sum Rate approach

Sum rate algorithm which we have implemented in this paper is an extension of an algorithm published in [11]. This algorithm is for IFC which directly cannot be extended to IFBC because IFBC is more complex than IFC. In IFBC because of large number of users its not logical to compute $U_k^{[l]}$ and $V_k^{[l]}$ as is done in [1].
We will compute $U_k^{[l]}$ only for first user and then we will continuously update it in each iteration as we did before. Algorithm is initialized in similar manner to the first case of orthogonal user selection computation ease of implementation is maintained by continuously updating $U_k^{[l]}$. The user subset is maintained and updated in each step. One user is selected every time and replaced with the one in the paper in such a way that sum rate is increased. This algorithm is implemented and results are shown for it. One good thing which this algorithm posses is that it is extendable to the currently existing algorithm. There are some good results which can be achieved using less number of antennas for the same algorithm. This algorithm appears to have results better than the rest of the algorithms and is linear in complexity but with a higher slope.

IV. Complexity Analysis

In results we will try the compute and then compare the algorithmic computational complexity of various algorithms we are going to implement i.e.brute force,s-orthogonalization and o-orthogonalization.We will plot and compare the sum rates at various SINR's vs number of users.The last plot will

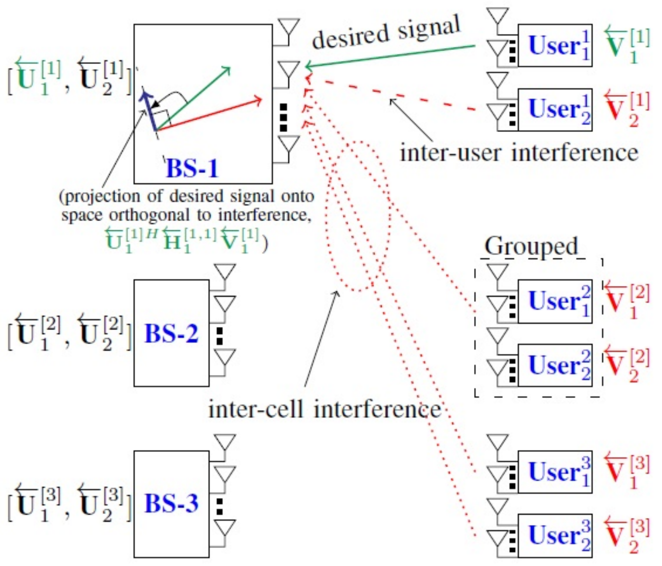

Fig. 2. Sum rate versus number of users in each cell when M = 6,N = 4,K = 2, L = 2 and ds = 2.

be used to depict the complexity(flops) vs number of users. I am not be able to complete the implementation of the project but i have done more than half of it by implementing brute force algorithm and to some extent orthogonality based approach. I have solved these two algorithms individually but have not summarized the whole results in a single window. The complexity of the algorithm is compute using flop counts. The complexity of an operation is counted as total number of flops in operation and is denoted byψ. Assume that the number of user in each in each cell are $K_l = K_T$

Complexity Analysis :	
Operation	Flop Count
Real Addition	one
Real Multiplication	one
Complex Addition	two
Complex Multiplication	six
$\|\mathbf{H}\|_F$	4MN
GSO(H)	$8N^2$M-2MN
SVD(H)	24N$M^2+48N^2M+54N^3$

A. Orthogonality Approach:

The initialization of the algorithm in Table-1 requires $K_T \times L$ frobenius norm computations and total flops required are $K_T \times L \times 4MN$. Let ψ_U denotes the flops required to compute the receiver beamforming matrix$\mathbf{U}_k^{[l]}$. for all users in the i_{th} cell. The computation of $\mathbf{U}_k^{[l]}$ from (11) requires SVD computation of $KM \times [M + KN]$ matrix, hence $\psi_U = \psi_{SVD}(KM, M + KN)$. The decoupled approach is effective in complexity reduction for K ¿ 3 and flops required are

$$\psi_U = K \times \psi_{SVD}(M, M+N) + \sum_{i=1}^{\lceil \log K \rceil} (\{ \left\lceil \frac{K}{2^i} \right\rceil \times (\psi_{SVD}(M, 2^{i-1}N - s_iM) + 8M(2^{i-1}N - s_iM)(2^iN - s_{i+1}M)) + K \times 8N(2^{i-1}N - s_iM)(2^iN - s_{i+1}M))\} \quad (27)$$

where $s_1 = 0$, $s_i = 2s_{i-1} + 1$and $\lceil a \rceil$is the smallest integer number greater than or equal to a.

Complexity Analysis :	
Operation	Flop Count
Generator Matrix A_G	$8MNd_s$
Generator Matrix B_G	[K(L-1)-1]×$8MNd_s$
GSO(A_G)	$8M^2d_{s-2Md_s}$
GSO(A_G)	$8M^2(K(L-1)d_s) - 2M(K(L-1)d_s)$
$A_GA_G^H$	$8M^2d_s$
$B_GB_G^H$	$8M^2(K(L-1)d_s)$
$\|A_GA_G^H - B_GB_G^H\|_F$	$6M^2$

The flops required to compute the sumrate R_p are ignored. The total flops of the algorithm is given by

$$\psi_{cho} \approx 4K_TLMN + L\psi_U + \{\psi_U + 8M^2d_s - 2Md_s + 8MNd_s \times [K(L-1)] + 8M^2(K(L-1)d_s) - 2M(K(L-1)d_s) + 8M^2d_s + 8M^2(K(L-1)d_s) + 6M^2\} \times (K_T - K + 1)KL \quad (28)$$

and hence the complexity varies linearly with the number of users in each cell(K_T)

B. Sum Rate Approach:

The flops required in initialization in table-2 are similar to previous algorithm, $K_TL \times 4MN$. The flops required to compute the receive beamforming matrices in a particular cell are ψ_U., like in the previous algorithm. The transmit matrix for the kth user in the lth cell, $\mathbf{V}_k^{[l]}$ needs SVD computation of $M \times [K(L-1) \times d_s]$ matrix, hence flops required are $\psi_{SVD}(M, K(L-1) \times d_s)$. To compute the pre-whitening filter $\mathbf{W}_k^{[l]}, 8d_s^2N$ flops are required for matrix multiplication. The complexity of inverse of $d_s \times d_s$ matrix is ignored. The computation of sum rate using (9) involves the multiplication $\mathbf{W}_k^{[l]}\bar{\mathbf{H}}_k^{[l,l]}$, complexity of which is $8NMd_s + 8Md_s^2 + 8d_s^3$. The flops required by the water-filling over d_s eigenmodes are ignored since d_s is smaller than M and N. Therefore, the total flops of the algorithm are

$$\psi_s \approx 4K_TLMN + L\psi_U + \psi_U + KL \times [\psi_{SVD}(M, K(L-1)d_s) + (8d_s^2N + 8NMd_s + 8Md_s^2 + 8d_s^3)] \times (K_T - K + 1)KL \quad (29)$$

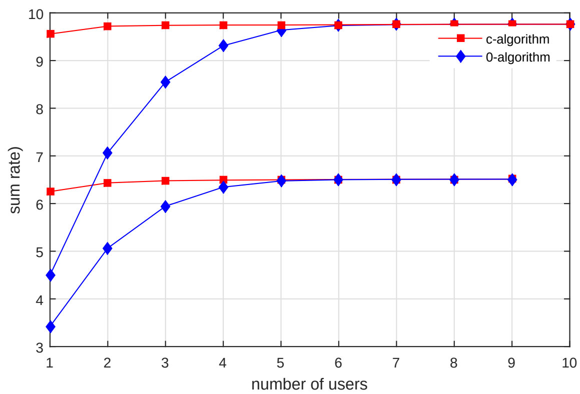

Fig. 3. Sum rate versus number of users in each cell when M = 3,N = 2,K = 2, L = 2 and d_s = 1.

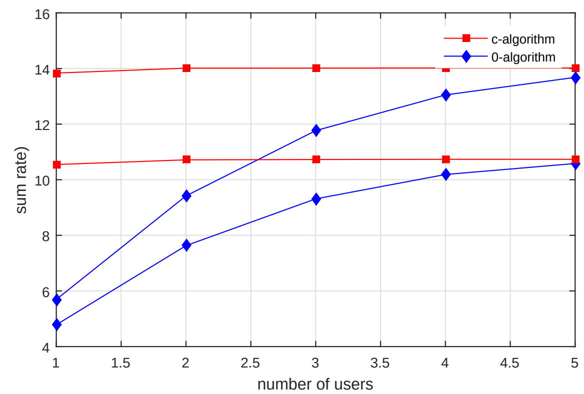

Fig. 4. Sum rate versus number of users in each cell when M = 6,N = 4,K = 2, L = 2 and d_s = 2

C. Brute-force Approach:

The flop count for brute-force selection algorithm to obtain the optimal solution can be written as

$$\begin{aligned}\psi_{opt} \approx & \left[\binom{K_T}{K}\right]^L \times \{KL \times \psi_{SVD}(M, K(L-1)d_s) \\ & + L\psi_U + KL \times (8d_s^2 N + 8NMd_s + 8Md_s^2 + 8d_s^3)\} \\ & \approx \mathcal{O}(K_T^{KL} K^{-KL-\frac{L}{2}+1} M^3 L)\end{aligned} \tag{30}$$

where the flops count ψ_U is determined for$K \leq 3$ as an example to demonstrate the complexity order. The order is shown to be exponential inK_T and we have used the Stirling's approximation to the factorial and approximated the binomial coefficient as

$$\binom{K_T}{K} \approx K_T^K K^{-K-\frac{1}{2}} \tag{31}$$

V. SIMULATION RESULT:

In this section, we provide the sum rate and flop count results for the orthogonality approach (o-algorithm) and sum rate approach (s-algorithm) and compare them with the brute-force selection algorithm. The sum rate results are averaged over 1000 random channel realizations. We will assume that the number of users in each cell $K_l = K_T, \forall l$. The total transmit power of each BS is fixed at P i.e. $P_l = P, \forall l$. The simulation results are shown for different values of total transmit power to noise variance ratio (SNR $= \frac{P}{\sigma^2}$) in dB.

It can be observed that the sum rate achieved by the two suboptimal algorithms namely s-algorithm and o-algorithm is more than 90% of the optimal sum rate achieved by the brute-force selection algorithm. The reduction in achievable sum rate in these suboptimal algorithms is because the search range of users is reduced. However, this reduction in search range has a significant impact on complexity. Thus, as we can see from (25), the complexity of brute-force search is exponential with respect to KT as compared to linear for the above suboptimal algorithms.

Further, in Fig. 3 and Fig. 4 we plot the user selection algorithm which selects a single user in each cell in IFC. . The achievable dof in IFBC in Fig. 3 (Fig. 4) are $d_s = 1(2)$ for each user making a total of 4 (8) dof while in IFC the total is $\min\{2M, 2N, \max(M, N)\}$ which is equal to 3 (6) dof. This explains the significant improvement in the sum rate when multiple users are selected than single user selection. In Fig. 5 the flop count of the two suboptimal algorithms for multi-user selection and of the algorithm for single user selection is compared as a function of the number of users in each cell (KT). Since $K \leq 3$, the $\mathbf{U}_k^{[l]}$ is computed using (11) and the flop count U will be used in (23), (24) accordingly. It can be seen that the total flop count of the o-algorithm is nearly half of the total flop count of s-algorithm. The reduction in complexity is because the sum rate computation in each step of the s-algorithm requires two SVD computation, one for $\mathbf{U}_k^{[l]}$ and other for $\mathbf{V}_k^{[l]}$, however, in o-algorithm the computation of $\mathbf{V}_k^{[l]}$ is not required. The computation of chordal distance is much less complex as compared to SVD computation, and this computation gain increases with increase in number of antennas.

VI. CONCLUSION:

The difference between the sum rate achieved by the s-algorithm and the o-algorithm becomes nearly constant as K_T increases. However, from Fig. 5 we can see that difference between the flop count of these algorithms increase with K_T. So o-algorithm is preferable when the number of users in each cell is large. The user selection problem has been addressed to improve the achievable sum rate of the MIMO-IFBC system. A suboptimal user selection algorithm is proposed to reduce the complexity of selection process. The algorithm exploits network reciprocity concepts and orthogonality between the desired signal space and interference space in the reciprocal system to select the users. An existing suboptimal algorithm based on the sum rate criteria is also extended to MIMO-IFBC. Simulation results show that the sum rate achieved by the orthogonality based algorithm and the extended sum rate based algorithm is close to the optimal sum rate. The complexity of these algorithms turns out to be linear with respect to the number of users in each cell as compared to exponential for brute-force search.

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VII. Appendix

TABLE I SUM RATE BASED LINEAR SEARCH ALGORITHM

1) Initialization: Define $\mathcal{T}^{[l]} = 1, ..., K_L$ for each $1 \leq l \leq L$, initialize the user subsets as
$\mathcal{S}^{[l]} = arglist_K max_{j \in \mathcal{T}^{[l]}} ||H_j^{[l,l]}||$ for each $1 \leq l \leq L$ such that $\mathcal{S}^{[l]} = s_1^{[l]}, ..., s_K^l; C = 0$. Perform the grouping and compute the initial value of receiver matrices $U_i^{[l]}, \forall i \in \mathcal{S}^{[l]}, \forall l$

2) for $l = 1 : L$
for $k = 1 : K$
For every $j \in \mathcal{T}^{[l]} s_1^{[l]}, ..., s_K^l$
   a) define $\mathcal{S}_{k,j}^{[l]temp} = \mathcal{T}^{[l]} | s_k^l = j$.
   b) Compute the temporary receiver matrix for the users in $\mathcal{S}_{k,j}^{[l]temp}$ using grouping as $U_j^{[l]temp}$.
   c) Using the $U_j^{[l]temp}$ and U_i^m, i $\in \mathcal{S}^{[l]} \forall m \neq l$ compute the transmit processing matrices using (12) as $V_i^m, i \in \mathcal{S}^{[m]} \forall m \neq l$ and $i \in \mathcal{S}_{k.j}^{[l]temp}$, for m = l.
   d) Using the computed values of receive and transmit matrices compute $R_j = R(\mathcal{S}^{[1]}, ..., \mathcal{S}^{[l1]}, \mathcal{S}_{k,j}^{[l]temp}, \mathcal{S}^{[l+1]}, ..., \mathcal{S}^{[L]})$ using (13) for the selected users.
   p = $argmax_{j \in \mathcal{T}^{[L]} - s_1^{[l]}, ..., s_K^l} \mathcal{R}_j$
   if $\mathcal{R}_P > \mathcal{C}$
   $\mathcal{C} \leftarrow \mathcal{R}_P$
   $U_i^m \leftarrow U_j^{[l]\text{temp}}, \forall i | in \mathcal{S}_j^{[l]temp}$
   $\mathcal{S} \leftarrow \mathcal{S}_j^{[l]temp}$

TABLE II ORTHOGONALITY BASED LINEAR SEARCH ALGORITHM

1) Initialization: Define $\mathcal{T}^{[l]} = 1, ..., K_L$ for each $1 \leq l \leq L$, initialize the user subsets as
$\mathcal{S}^{[l]} = arglist_K max_{j \in \mathcal{T}^{[l]}} ||H_j^{[l,l]}||$ for each $1 \leq l \leq L$ such that $\mathcal{S}^{[l]} = s_1^{[l]}, ..., s_K^l; C = 0$. Perform the grouping and compute the initial value of receiver matrices $U_i^{[l]}, \forall i \in \mathcal{S}^{[l]}, \forall l$

2) for $l = 1 : L$
for $k = 1 : K$
For every $j \in \mathcal{T}^{[l]} s_1^{[l]}, ..., s_K^l$
   a) define $\mathcal{S}_{k,j}^{[l]temp} = \mathcal{T}^{[l]} | s_k^l = j$.
   b) Compute the temporary intersection subspace and receiver matrix for the user in $S_k^{[l]temp}$ using grouping as $G_L^{temp]}$ and $U_i^{[l]temp}$,$\forall i \in \mathcal{S}_{k,j}^{[l]temp}$
   c) Compute generator matrix for the desired signal space as $A_G = [H_j^{[l,l]} U_j^{[l]}]_0$ for intersection space as $B_G = [G_l^{[temp]} \quad H^{[l,l]H}_{t(t \in \mathcal{S}_{k,j}^{[l]temp} - j)} \quad U^{[l]H}_{t(t \in \mathcal{S}_{k,j}^{[l]temp} - j)}$
   Using the computed values of receive and transmit matrices compute $R_j = R(\mathcal{S}^{[1]}, ..., \mathcal{S}^{[l1]}, \mathcal{S}_{k,j}^{[l]temp}, \mathcal{S}^{[l+1]}, ..., \mathcal{S}^{[L]})$ using (13) for the selected users.
   p = $argmax_{j \in \mathcal{T}^{[L]} - s_1^{[l]}, ..., s_K^l} ||A_G.A_G^H - B_G.B_G^H||_F$
   if $\mathcal{R}_P > \mathcal{C}$

$\mathcal{C} \leftarrow \mathcal{R}_P$
$U_i^m \leftarrow U_j^{[l]\text{temp}}, \forall i | in \mathcal{S}_j^{[l]temp}$
$\mathcal{S} \leftarrow \mathcal{S}_j^{[l]temp}$