A Comparative Analysis of Detection Filters in the Doctrine of MIMO Communications Systems

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***Abstract*—In this paper, we examine and compare various contemporary signal detection schemes for overloaded multiple-input multiple-output (MIMO) systems, where the number of receive antennas is less than that of transmitted streams as well as non over-loaded multiple-input multiple-output (MIMO) systems, where the number of transmit antennas is less than that of received streams. We try to formulate the signal detection as a convex optimization problem. In method 1: we analysed the two suboptimal detection receiving filter which are well researched and understood : *the receive zero-forcing filter* (RxZF), and the *the receive Weiner Filter* (RxWF) where we give close form solution and present a fundamental result that receive Weiner Filter (RxWF) converges to the zero-forcing filter for high signal-to-noise ratio. In method 2:, we elaborate an efficient approximation of the maximum likelihood (ML) detector for quadrature phase shift keying (QPSK) which is based on a convex relaxation of the ML problem. In method 3:, we uses the idea of the sum-of-absolutevalue (SOAV) optimization, where we formulate the signal detection as a convex optimization problem and solve it via a fast algorithm based on Douglas-Rachford splitting. Contrary to this, we uses an iterative approach to solve the optimization problem so as to improve the performance, with weighting parameters update in a cost function. Finally Simulation results compares the bit error rate (BER) performance of all the schemes for overloaded as well as non overloaded MIMO systems under white as well as coloured noise scenario.**

***Index Terms*—Overloaded MIMO Systems, Weiner Filtering, semidefinite relaxation, proximal splitting methods, Douglas-Rachford algorithm, SOAV optimization.**

I. INTRODUCTION

The multiple-input multiple-output (MIMO) communication systems have arise in many modern communication teech-nology, such as multi-user communication, Massive MIMO Systems, cooperative networks and multiple antenna channels. It is well known that the use of multiple transmit and receive antennas offers substantial gains to the system in comparison to the traditional single antenna systems. The surge in interest has occured in order to exploit these gains and to develop a robust system that must be able to efficiently detect the transmitted symbols at the receiver. Hence, detection in MIMO systems is one of the fundamental problems in state-of-the-art communication systems. Also for MIMO systems, low complexity signal detection method is essential because the required computational complexity of MIMO signal detection generally increases along with the increase of the antennas.

Existing literature on signal detection techniques can be classified into sub-optimal or quasi-optimal MIMO detection filters, such as the linear receivers, i.e, the zero-forcing (ZF) which removes interference, and the minimum mean squared error (MMSE) MIMO detectors which finds a tradeoff between noise and interference. [**?**] And maximum likelihood (ML) detector which is an optimal algorithm in the sense of minimum joint probability of error for detecting all the symbols simultaneously.

The task of detection filters is to remove the distortions generated by the channel and the perturbation caused by the noise. One key assumption in the case of receive processing is that the receiver knows the adopted signal processing at the transmitter which turns out to be the major drawback of receive filters as it increases the complexity of the receiver, because channel estimation and adaptation of the receive filter is necessary. For example, in the uplink of cellular mobile radio systems, receive processing is advantageous, because the complexity resides at the base station (BS). On the other hand, in the downlink, receive processing leads to more complex mobile stations (MSs).

One of the most promising suboptimal detection strategies is the semidefinite relaxation (SDR) detector. The SDR attempt to approximate the solution for the ML problem using a convex program that can be efficiently solved in polynomial time. The usual approach of the SDR problem is first to formulate the ML problem in a higher dimension and then relax the non-convex constraints; such relaxation will result in a semi definite program (SDP), for which there are efficient tools to obtain solutions in polynomial time. The success of SDR in demodulating BPSK signaling motivated its generalization to higher constellations, In this article we uses quadrature phase shift keying (QPSK) constellation set.

In this paper, we examine another signal detection scheme with much lower complexity than that of conventional schemes where we formulate the signal detection problem as a convex optimization problem, and the idea is based on the sum-of-absolute-value (SOAV) optimization, which is a technique to reconstruct a discrete-valued vector from its linear

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measurements. The optimization problem can be efficiently solved with proximal splitting methods even for underdetermined(overloaded) systems. To improve the performance, we extend SOAV optimization to weighted-SOAV optimization, where the prior information about the discrete-valued vector can be used, and propose an iterative approach, named iterative weighted-SOAV (IW-SOAV), using the estimate in the previous iteration as the prior information. Since the weighted-SOAV optimization problem can also be efficiently solved with proximal splitting methods, IW-SOAV can detect the transmitted signals with low computational complexity. Simulation results show that IW-SOAV can achieve much better bit error rate (BER) performance than conventional signal detection schemes.

II. METHOD 1:RECEIVE FILTERS

A. System Models And Assumptions:

Consider a MIMO system as depicted in (Fig. 1) which consists of the transmit filter $\boldsymbol{P}$, the channel $\boldsymbol{H}$, and the receive filter $\boldsymbol{G}$. We assume that $\boldsymbol{H} \in \mathbb{C}^{M\times N}$ is tall or square $(M \geq N)$ for receive processing and wide or square $(M \leq N)$ for transmit processing. Moreover, the number of information symbols B does not exceed min(M,N). If we consider receive processing, the signal processing $\boldsymbol{P} \in \mathbb{C}^{N\times B}$ at the transmitter is a priori known to the receiver and the chain $\boldsymbol{HP} \in \mathbb{C}^{M\times B} of \boldsymbol{P}$ and $\boldsymbol{H}$ has full rank, i.e., rank($\boldsymbol{HP}$)=B. Accordingly, the filter $\boldsymbol{G} \in \mathbb{C}^{B\times M}$ at the receiver is a priori known to the transmitter in the case of transmit processing and $\boldsymbol{GH} \in \mathbb{C}^{B\times N}$ has full rank, that is, rank($\boldsymbol{GH}$)=B. The system model, as well as the derivations presented in this article, are applicable to systems with flat fading and frequency selective fading channels. The transmitted signal $\boldsymbol{y}$ is the desired signal $\boldsymbol{y} \in \mathbb{C}^{B}$ transformed by the transmit filter (cf. Fig. 1)

$$\boldsymbol{y} = \boldsymbol{Ps} \in \mathbb{C}^{N} \tag{1}$$

where we assume that the average transmit power is fixed

$$\mathrm{E}\left[\|\mathbf{y}\|_2^2\right] = \mathrm{E}\left[\|\boldsymbol{Ps}\|_2^2\right] = \mathrm{tr}(\boldsymbol{PR_sP}^{\mathrm{H}}) = E_{\mathrm{tr}}. \tag{2}$$

After transmission over the channel $\boldsymbol{H}$, the received signal is perturbed by the noise $\boldsymbol{\eta} \in \mathbb{C}^{M}$ and passed through the receive filter $\boldsymbol{G}$ to obtain the estimate

$$\tilde{\boldsymbol{s}} = \boldsymbol{G}(\boldsymbol{HPs} + \boldsymbol{\eta}) \in \mathbb{C}^{B}. \tag{3}$$

Note that we assume that the noise is uncorrelated with the symbols, that is, $\mathrm{E}[\boldsymbol{\eta s}^{\mathrm{H}}] = \mathbf{0}_{M\times B}$.

B. Receive Zero-Forcing Filter (RxZF):

One type of linear receive processing arises from the constraint that $\tilde{\boldsymbol{s}}$ is an interference-free estimate of $\boldsymbol{s}$. Thus, we have to fulfill following equation [see (3)]:

$$\tilde{\boldsymbol{s}}\,|_{\boldsymbol{\eta}=\mathbf{0}_M} = \boldsymbol{GHPs} \equiv \boldsymbol{s}. \tag{4}$$

Since $\boldsymbol{s}$ is arbitrary and unknown to the receiver, the chain of the transmit filter $\boldsymbol{P}$, the channel $\boldsymbol{H}$, and the receive filter $\boldsymbol{G}$ must result in an identity mapping

$$\boldsymbol{GHP} = \mathbf{1}_B. \tag{5}$$

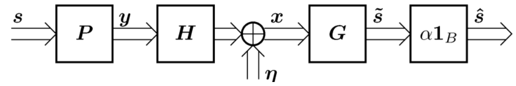

Fig. 1.

Note that this constraint can be fulfilled, because we assumed rank($\boldsymbol{HP}$)=B. With the above constraint and (3), the MSE of the RxZF (without the scalar Wiener filter of Fig. 1) can be shown to be the noise power at the filter output

$$\mathrm{E}\left[\|\boldsymbol{s} - \tilde{\boldsymbol{s}}\|_2^2\right] = \mathrm{E}\left[\|\boldsymbol{G\eta}\|_2^2\right]. \tag{6}$$

The RxZF minimizes the above MSE and removes the interference [cf. (8)]

$$\boldsymbol{G}_{\mathrm{ZF}} = \mathrm{argmin}_{\boldsymbol{G}}\mathrm{E}\left[\|\boldsymbol{G\eta}\|_2^2\right] \quad \text{s.t.: } \boldsymbol{GHP} = \mathbf{1}_B. \tag{7}$$

With the Lagrangian multiplier method (e.g., [77]), we obtain the RxZF

$$\boldsymbol{G}_{\mathrm{ZF}} = \left(\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{HP}\right)^{-1}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1} \in \mathbb{C}^{B\times M}. \tag{8}$$

C. Receive Wiener Filter (RxWF):

The RxWF minimizes the MSE without an additional constraint [see also (3)]

$$\boldsymbol{G}_{\mathrm{WF}} = \mathrm{argmin}_{\boldsymbol{G}}\mathrm{E}\left[\|\boldsymbol{s} - \tilde{\boldsymbol{s}}\|_2^2\right]. \tag{9}$$

After setting the derivative of the MSE to zero, we yield

$$\boldsymbol{G}_{\mathrm{WF}} = \left(\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{HP} + \boldsymbol{R}_{\boldsymbol{s}}^{-1}\right)^{-1}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1} \tag{10}$$

where we utilized the matrix inversion lemma (e.g., [79]). Equation (12) helps to understand the dependence of the RxWF on the SNR. The second summand can be neglected for high SNR and the RxWF converges to the RxZF [cf. (10)] MSE of the given MIMO system can be expressed as:

$$\varepsilon = \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2\right] = \mathrm{E}\left[\|\boldsymbol{s} - \alpha\tilde{\boldsymbol{s}}\|_2^2\right] \in \mathbb{R}_{0,+} \tag{11}$$

We need a scalar Wiener filter α at the end of the filter chain to get a reasonable comparison. The scalar Wiener filter Îś minimizes the MSE $\mathrm{E}[\|\boldsymbol{s} - \alpha\tilde{\boldsymbol{s}}\|_2^2]$ of (4) and is found in a similar way as $\boldsymbol{G}_{\mathrm{WF}}$. We obtain for the scalar Wiener filter

$$\alpha = \frac{\mathrm{tr}\left(\boldsymbol{R_sP}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{G}^{\mathrm{H}}\right)}{\mathrm{tr}\left(\boldsymbol{G}\left(\boldsymbol{HPR_sP}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{R_\eta}\right)\boldsymbol{G}^{\mathrm{H}}\right)} \in \mathbb{C} \tag{12}$$

III. METHOD 2:SEMI DEFINITE RELAXATION

One of the most promising suboptimal detection strategies is the semidefinite relaxation (SDR) detector, which recently gained considerable attention. The main reason for the high computational complexity of the ML detector is due to the fact that it is a non convex optimization problem. SDR is an attempt to approximate it using a convex program that can be efficiently solved in polynomial time.

A. *System Models And Assumptions:*

Consider the standard MIMO channel

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\overline{\mathbf{s}} + \overline{\mathbf{w}} \tag{13}$$

where $\overline{\mathbf{y}}$ is the received signal of length N, $\overline{\mathbf{H}}$ is an $N \times K$ channel matrix, $\overline{\mathbf{s}}$ is the length K vector of transmitted symbols, and $\overline{\mathbf{w}}$ is a length N complex normal zero-mean noise vector with covariance $\sigma^2\mathbf{I}$. The symbols of $\overline{\mathbf{s}}$ belong to some known complex constellation. In this article, we consider the QPSK constellation, i.e., the real part and the imaginary part of $\overline{\mathbf{s}}_i$ for i=1,. . ., K belong to the set $\{\pm 1\}$.

In order to avoid the need to handle complex-valued variables, it is customary to use the following decoupled model:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{14}$$

where

$$\boldsymbol{y} = \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{y}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{y}}\} \end{bmatrix}, \boldsymbol{H} = \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{H}}\} & -\mathrm{Im}\{\tilde{\boldsymbol{H}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{H}}\} & \mathrm{Re}\{\tilde{\boldsymbol{H}}\} \end{bmatrix},$$
$$\boldsymbol{s} = \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{s}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{s}}\} \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{v}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{v}}\} \end{bmatrix}.$$

Using these definitions, the ML detector of the transmitted symbols is

$$\begin{cases} \min_{\mathbf{s}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 \\ \text{s.t.} \quad \mathbf{s}_i \in \{\pm 1\}, \quad i = 1, \dots, 2K. \end{cases} \tag{15}$$

The program ML is a combinatorial problem and can be solved by searching over all of the 4^K possibilities. Clearly, as K increases, this option becomes impractical.

B. *SDR via Rank Relaxation:*

The key observation that leads to the SDR is that the constraint $\mathbf{s}_i \in \{\pm 1\}$ for $i = 1, \dots, 2K$ can be expressed as

$$(s_i + 1)(s_i - 1) = 0, \quad i = 1, \dots, 2K. \tag{16}$$

We can use eqn.(16) to rewrite the problem ML as

$$\begin{cases} \min_{\mathbf{s}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 \\ s.t \;\; s_i^2 - 1 = 0, \quad i = 1, \dots, 2K \end{cases} \tag{17}$$

The next step in deriving the SDR is formulating the optimization problem in a higher dimension. We replace the vectors $\boldsymbol{s}$ with a rank-one semidefinite matrix $\boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^T$, where

$$\boldsymbol{w}^T = \begin{bmatrix} s^T & 1 \end{bmatrix} \tag{18}$$

Using this change of variables, we can easily identify $\mathbf{W}_{1,1} = \mathbf{s}\boldsymbol{s}^T$, $\boldsymbol{W}_{2,2} = \mathbf{1}$, $\boldsymbol{W}_{1,2} = \boldsymbol{s}$, and $\boldsymbol{W}_{2,1} = \boldsymbol{s}^T$, where $\boldsymbol{W}_{i,j}$, for i,j=1, 2, are the (i,j)th sub-blocks of $\boldsymbol{W}$ of appropriate sizes. Therefore, problem in (17) is equivalent to

$$\begin{cases} \min_{\mathbf{W}} \mathrm{Tr} \left\{ \mathbf{W} \begin{bmatrix} \mathbf{H^T H} & 0 & -\mathbf{H^T y} \\ 0 & 0 & 0 \\ -\mathbf{y^T H} & 0 & \mathbf{y^T y} \end{bmatrix} \right\} \\ \text{s.t.} \;\; \mathrm{diag}\{\mathbf{W}_{1,1}\} - \mathbf{W}_{2,2} = \mathbf{0} \\ \mathbf{W} \succeq \mathbf{0} \\ \mathbf{W_{2,2}} = \mathbf{1} \\ rank(\mathbf{W}) = \mathbf{1} \end{cases} \tag{19}$$

The above program is not convex because of the rank-one constraint. Dropping this constraint results in the SDR

$$\begin{cases} \min_{\mathbf{W}} \mathrm{Tr} \left\{ \mathbf{W} \begin{bmatrix} \mathbf{H^T H} & 0 & -\mathbf{H^T y} \\ 0 & 0 & 0 \\ -\mathbf{y^T H} & 0 & \mathbf{y^T y} \end{bmatrix} \right\} \\ \text{s.t.} \;\; \mathrm{diag}\{\mathbf{W}_{1,1}\} - \mathbf{W}_{2,2} = \mathbf{0} \\ \mathbf{W} \succeq \mathbf{0} \\ \mathbf{W_{2,2}} = \mathbf{1} \end{cases} \tag{20}$$

Note that the SDR has a linear objective subject to affine equalities and a linear matrix inequality. Such problems are known as SDP and can be efficiently solved in polynomial time. If the optimal argument $\mathbf{W}$ of SDR has rank one, then the relaxation is tight, and the ML solution of $\mathbf{s}$ is the first 2K elements of the last column of $\mathbf{W}$. Otherwise, SDR is only an approximation of ML, and there is no strict relation between $\mathbf{W}$ and $\mathbf{s}$. Instead, there are a few standard techniques for approximating $\mathbf{s}$ based on $\mathbf{W}$

- **Singularvalue decomposition:** Let $\mathbf{u}$ denote the eigenvector of associated with its maximal eigenvalue. Then

$$\widehat{s}_i = \text{quantize}\left(\frac{\mathbf{u}_i}{\mathbf{u}_{2K+1}}\right), \quad i = 1, \dots, 2K. \tag{22}$$

where $I_{\tilde{k},m} = \sqrt{L}\acute{I}_{\tilde{k},m}$, with average power $P_m = \acute{P}_m L$.

- **Randomization:** Let $\widetilde{\mathbf{W}} = \mathbf{V}^T\mathbf{V}$ be the Cholesky factorization of $\widetilde{\mathbf{W}}$ and denote the columns of $\mathbf{V}$ by $\mathbf{v}_i$. Then

$$\widehat{s}_i = \text{quantize}\left(\frac{\mathbf{v}_i^T\mathbf{r}}{\mathbf{v}_{2K+1}^T\mathbf{r}}\right), \quad i = 1, \dots, 2K \tag{23}$$

where r is a random vector uniformly distributed on a (2K+1)-dimensional unit sphere. In order to improve the approximation quality, the randomization is repeated a number of times, and the solution yielding the best objective value is chosen.

IV. METHOD 3:SOAV OPTIMIZATION

SOAV optimization is a technique to reconstruct an unknown discrete-valued vector as $\boldsymbol{x} = [x_1 \cdots x_N]^{\mathrm{T}} \in \{c_1, \dots, c_P\}^N \subset \mathbb{R}^N$ from its linear measurements $\boldsymbol{\eta} = \boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{A} \in \mathbb{R}^{M \times N}$. If we assume $\Pr(x_i = c_p) = 1/P$ $(p = 1, \dots, P)$ for all x_i $(i = 1, \dots, N)$, each of $\boldsymbol{x} - c_1\mathbf{1}, \dots, \boldsymbol{x} - c_P\mathbf{1}$ has approximately N/P zero elements. Based on this property and the idea of ℓ_1 optimization in compressed sensing, SOAV optimization solves

$$\begin{aligned} &\underset{\boldsymbol{x} \in \mathbb{R}^N}{\text{minimize}} \frac{1}{P} \sum_{p=1}^{P} \|\boldsymbol{x} - c_p\mathbf{1}\|_1 \\ &\text{subject to } \boldsymbol{\eta} = \boldsymbol{A}\boldsymbol{x} \end{aligned} \tag{24}$$

to reconstruct $\boldsymbol{x}$ from $\boldsymbol{\eta}$.

A. *Signal Detection via SOAV Optimization:*

In MIMO systems, the transmitted signal vector $\boldsymbol{s}$ is commonly discrete and the received signal vector $\boldsymbol{y}$ can be regarded as its linear observations if the noise can be ignored. Since each element of $\boldsymbol{s}$ is 1 or âĹŠ1 for the case with QPSK, we can formulate the signal detection problem as SOAV optimization, i.e.,

$$\begin{aligned}&\underset{\boldsymbol{z}\in\mathbb{R}^{2n}}{\text{minimize}}\ \frac{1}{2}\|\boldsymbol{z}-\mathbf{1}\|_1+\frac{1}{2}\|\boldsymbol{z}+\mathbf{1}\|_1\\ &\text{subject to } \boldsymbol{y}=\boldsymbol{H}\boldsymbol{z}.\end{aligned} \tag{25}$$

Since the received signal vector $\boldsymbol{y}$ contains the additive noise, we modify the optimization problem as follows:

$$\begin{aligned}\underset{\boldsymbol{z}\in\mathbb{R}^{2n}}{\text{minimize}}\ &\frac{1}{2}\|\boldsymbol{z}-\mathbf{1}\|_1+\frac{1}{2}\|\boldsymbol{z}+\mathbf{1}\|_1\\ &+\frac{\alpha}{2}\|\boldsymbol{y}=\boldsymbol{H}\boldsymbol{z}\|_2^2\end{aligned} \tag{26}$$

by using the idea of $\ell_1-\ell_2$ optimization. Here, $\alpha>0$ is a given constant. The solution of above minimization can be obtained with the following theorem.

• **Theorem**: Let $\phi_1,\phi_2:\mathbb{R}^{2n}\to(-\infty,\infty]$ be lower semi-continuous convex functions and $(\text{ri dom }\phi_1)\cap(\text{ri dom }\phi_2)\neq\emptyset$. In addition, $\phi_1(\boldsymbol{z})+\phi_2(\boldsymbol{z})\to\infty$ as $\|\boldsymbol{z}\|_2\to\infty$ is assumed. A sequence $\boldsymbol{z}_k(k=0,1,\ldots)$ converging to the solution of

$$\underset{\boldsymbol{z}\in\mathbb{R}^{2n}}{\text{minimize}}\quad \phi_1(\boldsymbol{z})+\phi_2(\boldsymbol{z}) \tag{27}$$

can be obtained by using the following Douglas-Rachford algorithm. Here, the proximity operator of a function $\phi:\mathbb{R}^{2n}\to\mathbb{R}$ is defined as

$$\text{prox}_{\phi}(\boldsymbol{z})=\arg\min_{\boldsymbol{u}\in\mathbb{R}^{2n}}\ \phi(\boldsymbol{u})+\frac{1}{2}\|\boldsymbol{z}-\boldsymbol{u}\|_2^2. \tag{28}$$

• Algorithm 1. *Douglas-rachford algorithm* 1)Fix $\varepsilon\in(0,1),\gamma>0$ and $\boldsymbol{r}_0\in\mathbb{R}^{2n}$ 2)For $k=0,1,2,\ldots$ iterate

$$\begin{cases}\boldsymbol{z}_k=\text{prox}_{\gamma\phi_2}(\boldsymbol{r}_k)\\ \lambda_k\in[\varepsilon,2-\varepsilon]\\ \boldsymbol{r}_{k+1}=\boldsymbol{r}_k+\lambda_k(\text{prox}_{\gamma\phi_1}(2\boldsymbol{z}_k-\boldsymbol{r}_k)-\boldsymbol{z}_k).\end{cases} \tag{29}$$

We can rewrite (8) as

$$\underset{\boldsymbol{z}\in\mathbb{R}^{2n}}{\text{minimize}}\quad f(\boldsymbol{z})+g(\boldsymbol{z}), \tag{30}$$

where $f(\boldsymbol{z})=\|\boldsymbol{z}-\mathbf{1}\|_1/2+\|\boldsymbol{z}+\mathbf{1}\|_1/2$ and $g(\boldsymbol{z})=\alpha\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{z}\|_2^2/2$. The proximity operators of $\gamma f(\boldsymbol{z})$ and $\gamma g(\boldsymbol{z})$ can be obtained as

$$[\text{prox}_{\gamma f}(\boldsymbol{z})]_j=\begin{cases}z_j+\gamma & (z_j<-1-\gamma)\\ -1 & (-1-\gamma\le z_j<-1)\\ z_j & (-1\le z_j\le 1)\\ 1 & (1\le z_j<1+\gamma)\\ z_j-\gamma & (1+\gamma\le z_j)\end{cases}, \tag{31}$$

and

$$\text{prox}_{\gamma g}(\boldsymbol{z})=(\boldsymbol{I}+\alpha\gamma\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})^{-1}(\boldsymbol{z}+\alpha\gamma\boldsymbol{H}^{\mathrm{T}}\boldsymbol{y}), \tag{32}$$

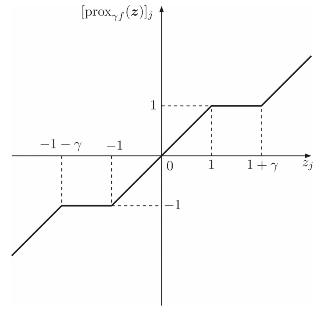

Fig. 2.

respectively, where $[\text{prox}_{\gamma f}(\boldsymbol{z})]_j$ $(j=1,\ldots,2n)$ represents the jth element of $\text{prox}_{\gamma f}(\boldsymbol{z})$. Note that $[\text{prox}_{\gamma f}(\boldsymbol{z})]_j$ is a function of z_j only as shown in Fig. 1. By solving (8) with the Douglas-Rachford algorithm, the estimate of the transmitted signal vector $\boldsymbol{s}$ can be obtained.

B. *An Iterative Approach: IW-SOAV*

Assuming that we have information on prior probabilities of $w_j^+=\Pr(s_j=1)$ and $w_j^-=\Pr(s_j=-1)$, we extend the problem of (8) to weighted-SOAV optimization problem as

$$\begin{aligned}\underset{\boldsymbol{z}\in\mathbb{R}^{2n}}{\text{minimize}}\quad &\sum_{j=1}^{2n}(w_j^+|z_j-1|+w_j^-|z_j+1|)\\ &+\frac{\alpha}{2}\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{z}\|_2^2.\end{aligned} \tag{33}$$

If there is no prior information about $\boldsymbol{s}$, i.e., $w_j^+=w_j^-=1/2$, the optimization problem (14) is equivalent to (8). If $w_j^+>w_j^-$ then $\arg\min_{z_j} f_{w_j}(z_j)=1$, where $f_{w_j}(z_j)=w_j^+|z_j-1|+w_j^-|z_j+1|$, thus the solution of z_j in (14) tends to take the value close to 1, and vice versa. The optimization problem (14) can also be solved by using the Douglas-Rachford algorithm. The proximity operator of

$$\gamma f_{\boldsymbol{w}}(\boldsymbol{z})=\gamma\sum_{j=1}^{2n}(w_j^+|z_j-1|+w_j^-|z_j+1|) \tag{34}$$

can be written as

$$\begin{aligned}&[\text{prox}_{\gamma f_{\boldsymbol{w}}}(\boldsymbol{z})]_j\\ &=\begin{cases}z_j+\gamma & (z_j<-1-\gamma)\\ -1 & (-1-\gamma\le z_j<-1-d_j\gamma)\\ z_j+d_j\gamma & (-1-d_j\gamma\le z_j<1-d_j\gamma)\\ 1 & (1-d_j\gamma\le z_j<1+\gamma)\\ z_j-\gamma & (1+\gamma\le z_j)\end{cases}\end{aligned} \tag{35}$$

as shown in Fig. 2, where $d_j=w_j^+-w_j^-$. By solving the optimization problem (14) via the Douglas-Rachford algorithm with $\text{prox}_{\gamma f_{\boldsymbol{w}}}$ and $\text{prox}_{\gamma g}$, a new estimate of the transmitted signal vector $\boldsymbol{s}$ can be obtained. The prior information on $\boldsymbol{s}$ is not available in a common scenario, however, assuming iterative approach, the estimate in the previous iteration can be used to obtain the prior probabilities. Specifically, in the

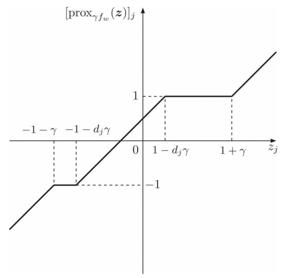

Fig. 3.

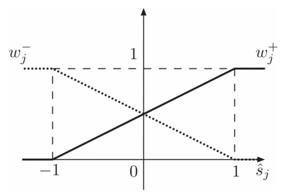

Fig. 4.

proposed iterative approach named IW-SOAV, we iteratively solve the weighted-SOAV optimization problem (14) while updating the parameters w_j^+ and w_j^- $(i = 1, \ldots, 2n)$ as

$$w_j^+ = \begin{cases} 0 & (\hat{s}_j < -1) \\ \frac{1+\hat{s}_j}{2} & (-1 \leq \hat{s}_j < 1) \\ 1 & (1 \ \leq \hat{s}_j) \end{cases} \quad (36)$$

and

$$w_j^- = 1 - w_j^+ = \begin{cases} 1 & (\hat{s}_j < -1) \\ \frac{1-\hat{s}_j}{2} & (-1 \leq \hat{s}_j < 1), \\ 0 & (1 \ \leq \hat{s}_j) \end{cases} \quad (37)$$

where $\hat{s}_j$ is the estimate of s_j in the previous iteration. Fig. 3 shows w_j^+ and w_j^- as a function of $\hat{s}_j$. w_j^+ is large when $\hat{s}_j$ is large, and w_j^- is large when $\hat{s}_j$ is small. This is because the estimates close to 1 or âĹŠ1 will be more reliable than those close to 0. The proposed algorithm of IW-SOAV is summarized as follows:

• Algorithm 2. *Signal Detection via IW-SOAV* 1) Let $\hat{s} = \mathbf{0}$ and iterate a)-c) for L times. a) Compute w_j^+,w_j^- with (17), (18). b) Fix $\varepsilon \in j \in (0, 1), \gamma > 0, K > 0$, and $\boldsymbol{r}_0 \in \mathbb{R}^{2n}$. c) For $k = 0, 1, 2, \ldots, K$, iterate

$$\begin{cases} \boldsymbol{z}_k = \text{prox}_{\gamma g}(\boldsymbol{r}_k) \\ \lambda_k \in [\varepsilon, 2 - \varepsilon] \\ \boldsymbol{r}_{k+1} = \boldsymbol{r}_k + \lambda_k(\text{prox}_{\gamma f_{\boldsymbol{w}}}(2\boldsymbol{z}_k - \boldsymbol{r}_k) - \boldsymbol{z}_k) \end{cases} \quad (38)$$

and let $\hat{\boldsymbol{s}} = \boldsymbol{z}_K$.

2) Obtain sgn($\hat{\boldsymbol{s}}$) as the final estimate of $\boldsymbol{s}$.

V. Simulation RESULTS:

In this section we compare the various detection filters by applying them to overloaded MIMO system(50×30) and non overloaded MIMO system (30×50) using computer simulations. The SOAV program was solveed using cvx package[5]. The Transmit filter P has identity mapping i.e. $\boldsymbol{P_{rx}} = \mathbf{1}_2$.

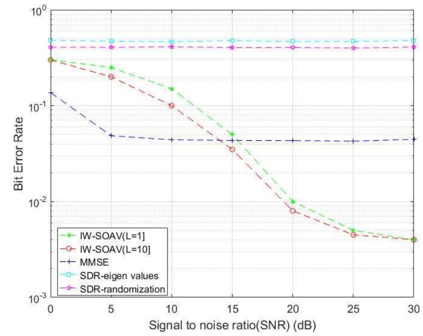

Fig. 5. BER V_s SNR plots for non-overloaded(30×50) MIMO systems under spatially coloured noise scenerio.

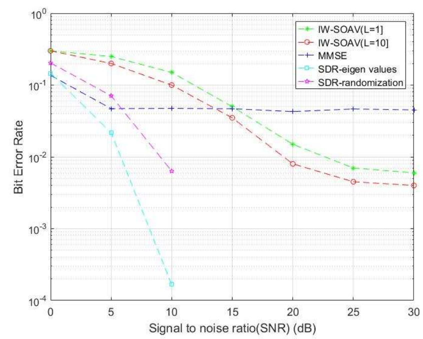

Fig. 6. BER V_s SNR plots for non-overloaded(30×50) MIMO systems under AWGN scenerio.

Per channel realization 100 QPSK symbols for each of the B=2 parallel data streams are transmitted, where we assume uncorrelated data streams and noise, i.e., $\boldsymbol{R_s} = \mathbf{1}_2$ and $\boldsymbol{R_\eta} = \sigma_{\boldsymbol{\eta}^2 \mathbf{1}_2}$. We set the transmit power to $E_{\text{tr}} = 2$, that is, unit transmit power is used for one symbol in the average. We assume uncorrelated Rayleigh fading and normalize the channel matrix such that $\mathrm{E}[\|\boldsymbol{H}\|_{\mathrm{F}}^2] = 1$. The transmitter knows the exact instantaneous channel state information.In the simulation for **method2** and **method3**, flat Rayleigh fading channels are assumed and $\tilde{\boldsymbol{H}}$ is composed of independent and identically distributed complex Gaussian random variables with zero mean and unit variance. Fig.6 and Fig.8 depicts the comparative analysis under spatially coloured noise, when the noise η has the covariance matrix

$$\boldsymbol{R_\eta} = \frac{\sigma_{\boldsymbol{\eta}}^2}{11} \begin{bmatrix} 11 & 9 \\ 9 & 11 \end{bmatrix}$$

It may be the case that the performance of the given schemes may vary on changing the number of antennas at the transmitter or receiver end. In **method3** The parameter α is selected as $\alpha = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1, 1$, and 1 for SNR per receive antenna of 0, 5, 10, 15, 20, 25, and 30 (dB), respectively. The other parameters of the proposed schemes are set as K=50, $\varepsilon = 0.1, \gamma = 1, \lambda_k = 1.9$ $(k = \ 0, 1, \ldots, K)$

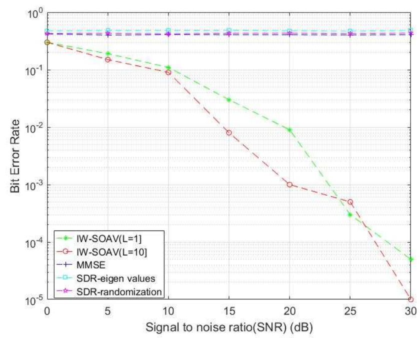

Fig. 7. BER V_s SNR plots for overloaded(50×30) MIMO systems under spatially coloured noise scenerio.

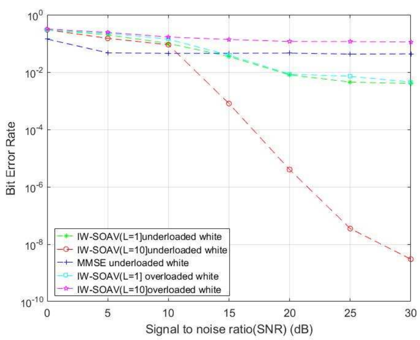

Fig. 8. BER V_s SNR plots for overloaded(50×30) MIMO systems under AWGN scenerio.

VI. Conclusion:

The design of robust detecting filters is an pervasive challenge in the field of communication systems. Hence in this section we try to present a comprehensible comparison between the various detection schemes which were discussed in the earlier sections.

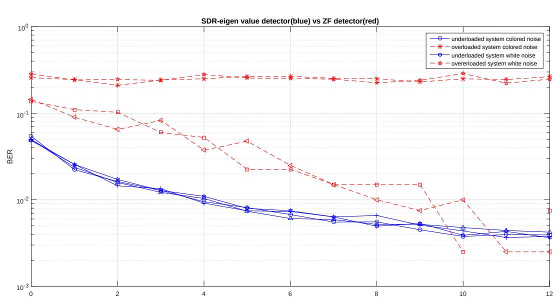

Fig. 9.

BER comparison under White noise scenario :

Name of Detection Technique	SNR=	SNR= 3	SNR=
Rx Zero Forcing Filter	AF	AFG	004
Rx Weiner Filter	AX	ALA	248
SDR via rank relaxation	AL	ALB	008
SDR via EVD	DZ	DZA	012
SOAV Optimization L=1	AS	ASM	016
SOAV Optimization L=10	AD	AND	020

BER comparison under coloured noise scenario :

Name of Detection Technique	SNR=	SNR= 3	SNR=
Rx Zero Forcing Filter	AF	AFG	004
Rx Weiner Filter	AX	ALA	248
SDR via rank relaxation	AL	ALB	008
SDR via EVD	DZ	DZA	012
SOAV Optimization L=1	AS	ASM	016
SOAV Optimization L=10	AD	AND	020

As it is apparent from the above table that